

Local Rules of Crystallinity and Groups in Delone Sets

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Introduced by B. Delone (Delaunay) in 1934 as (r, R) -systems and called Delone sets today, these point sets are used to model atomic structures of periodic crystals, non-periodic quasicrystals, glasses, and so on. Numerous applications of this concept are due to the simplicity and depth of its definition given by Delone. According to the definition, in a Delone set X ,

- (1) a real number $r > 0$ means that an every (open) ball of radius r contains at most one point of X ;
- (2) a real number $R > 0$ is such that every (closed) ball of radius R contains at least one point of X .

It is easy to see that Condition (1) can be reformulated as follows: the distance between any two points of the Delone set is not lesser than $2r$. Condition (2) is equivalent to the next one: the distance from an arbitrary point of space to the nearest point of X does not exceed R .

In the talk, we will discuss results on the foundations of geometric crystallography obtained in recent decades at school of Delone. The focus of this study is search for and proving local rules that guarantee Delone set to be a regular system (a set with point-transitive group and therefore periodic).

We will also discuss quite recent results and conjectures on local groups in arbitrary (without any additional conditions) Delone sets.