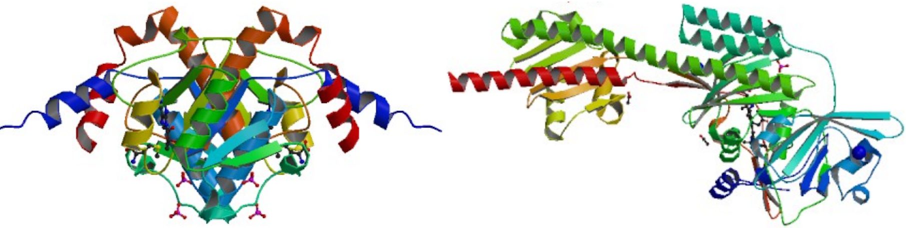
Visualizing knotted structures in 3-page books

Chris Smithers

Knotted structures in molecules

Protein Data Bank http://www.rcsb.org/pdb.

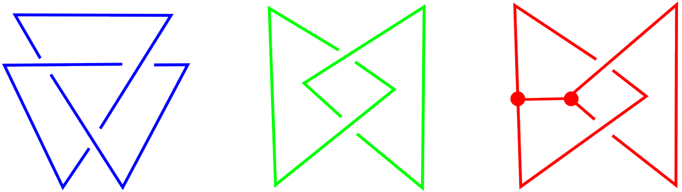


Left: 1v2x tRNA methyltransferase. Right: 3zq5.

- *Encode* long knotted structures in a simple way.
- *Compare* knotted structures up to deformations.

Polygonal knotted graphs in $\mathbb{R}^3$

Def: a knotted graph is an embedding $f : G \to \mathbb{R}^3$ consisting of finitely many *arcs*. So $f(G)$ has no self-intersections, but may have *crossings* when projected onto $\mathbb{R}^2$.

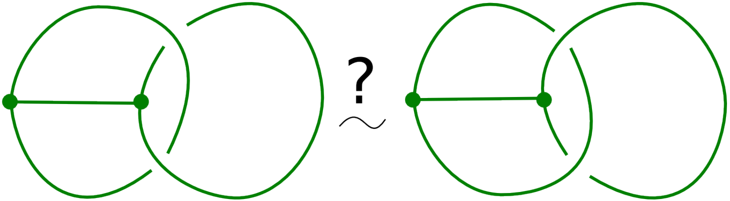


If $G \approx S^1$, the knotted graph $S^1 \subset \mathbb{R}^3$ is a knot.

If $G \approx \sqcup_{i=1}^m S_i^1$, the knotted graph is called a link.

Isotopy of knotted graphs

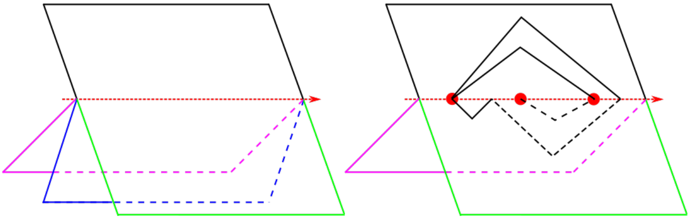
Def: an ambient isotopy between graphs $G, H \subset \mathbb{R}^3$ is a continuous family of ambient homeomorphisms $F_t : \mathbb{R}^3 \to \mathbb{R}^3$, $t \in [0, 1]$,
where $F_0 = \mathbf{id}$ on $\mathbb{R}^3$ and $F_1(G) = H$.



Question: When are two knotted graphs isotopic?

k-page books

Def: a k-page book is a collection of k half planes, with a common boundary line: the spine



We can talk about a k-page embedding of a knotted graph

The main result

Theorem [Kurlin, 2015]

There exists a 3-page embedding of a knotted graph in 3-pages

The main result

Theorem [Kurlin, 2015]

There exists a 3-page embedding of a knotted graph in 3-pages

We can actually achieve this in linear time, with respect to the number of edge segments

A brief overview

We break down the process into three short algorithms

A brief overview

We break down the process into three short algorithms

- Building a corresponding planar graph

A brief overview

We break down the process into three short algorithms

- Building a corresponding planar graph
- Embedding that planar graph appropriately

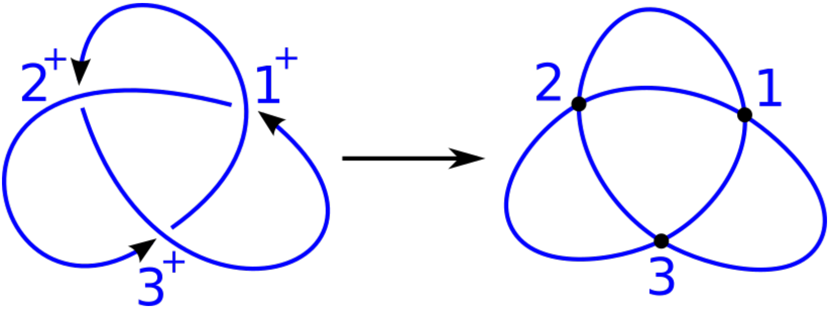
A brief overview

We break down the process into three short algorithms

- Building a corresponding planar graph
- Embedding that planar graph appropriately
- Using a third page to perform local upgrades

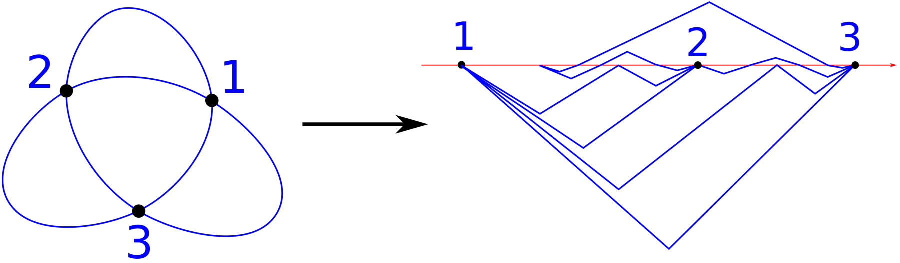
Build a planar graph

We "forget" about crossings

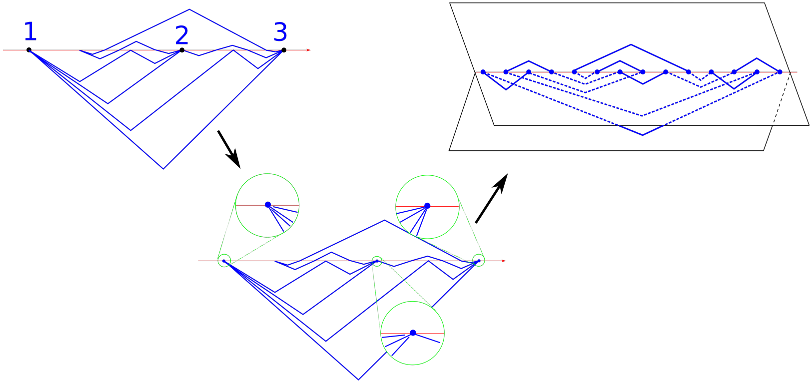


Embed the planar graph

We find a 2-page embedding of the planar graph



Perform Local Upgrades



Thanks!

Thank you all for listening