Random eigenvalues of graphenes and the triangulation of plane

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We analyse the numbers of closed paths of length $k \in \mathbb{N}$ on two important regular lattices: the hexagonal lattice (also called *graphene* in chemistry) and its dual triangular lattice. These numbers form a moment sequence of specific random variables connected to the distance of a position of a planar random flight (in three steps) to the origin. Here, we call such an random variable a *random eigenvalue* of the underlying lattice. Explicit formulae for the probability density and characteristic functions of these random eigenvalues are given for both the hexagonal and the triangular lattice. Furthermore, it is proven that both probability distributions can be approximated by a functional of the random variable uniformly distributed on increasing intervals $[0, b]$ as $b \to \infty$. This yields a simple way to simulate these random eigenvalues without generating graphene and triangular lattice graphs. To show that approximation, we first prove an interesting integral identity for a specific series containing the third powers of the modified Bessel functions I_n of nth order, $n \in \mathbb{Z}$. Such series play a crucial role in many contexts, in particular, in analysis, combinatorics and theoretical physics.

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